B. Sc. I Semester-I Subject-Electronics Paper-I

Network Theorems

Chapter :-I

§ Voltage divider theorem:

If a voltage divider circuit has N resistors (R_1, R_2, ... R_N) connected in series with source voltage V then resistor R_i will have voltage drop

$$V_i = \frac{R_i}{R_1 + R_2 + ... + R_N} V \quad (1)$$

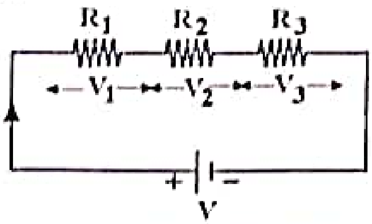

Fig. 1. Voltage divider circuit.

Consider the circuit shown in figure 1 in which three resistors R_1, R_2, R_3 are connected in series. Suppose V is the source voltage connected across the combination then total current I through the circuit is

$$I = \frac{V}{R_1 + R_2 + R_3} \quad (2)$$

The voltage drop across R_1 is then

$$V_1 = R_1 I \quad (4)$$

or

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} V \quad (5)$$

Similarly the voltage drops across R_2 and R_3 respectively are

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} V \quad (6)$$

and

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} V \quad (7)$$

§ Current divider theorem:

If a current divider circuit has N resistors (R_1, R_2, ...R_N) connected in parallel with source current I then current through resistor R_i will be

$$I_i = \frac{\frac{1}{R_i}}{\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}} I \quad (1)$$

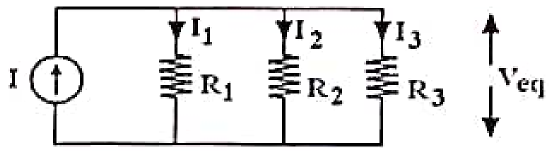

Fig. 1. Current divider circuit.

Consider the circuit shown in figure 1 in which three resistors R_1, R_2, R_3 are connected in parallel. Suppose I is the source current connected across the combination and Veq is the voltage drop across the combination then

$$V_{eq} = R_{eq} I \tag{2}$$

But $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{3}$$

Thus $$V_{eq} = \frac{I}{\left[\frac{1}{R_{eq}}\right]} = \frac{I}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \tag{4}$$

Therefore current through R_1 is

$$I_1 = \frac{V_{eq}}{R_1} \tag{5}$$

Using equations (4) in equation (5), we get

$$I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I \tag{6}$$

Similarly the currents through resistors R_2 and R_3 respectively are

$$I_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I \tag{7}$$

and $$I_3 = \frac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I \tag{8}$$

§ Ideal constant voltage source:

The voltage source whose output voltage remains constant whatever the change in the load resistor is called ideal constant voltage source. Such voltage source possesses zero internal resistance so that internal voltage drop in the source is zero. In practice, none such ideal constant voltage source can be constructed. However attempts are made to reduce the internal resistance of the source.

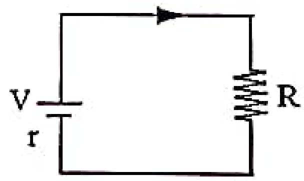

Fig. 1a.

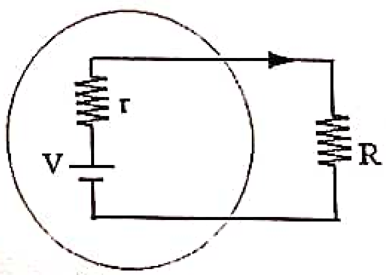

Fig. 1b.

Fig. 1a shows a resistor 'R' is connected across a voltage source. The internal resistance of a voltage source is always in series of the source. The equivalent circuit is shown in fig. 1b in which r indicates the internal resistance of the source. Therefore, the voltage drop across 'R' is $\frac{RV}{r+R}$ and can be close to 'V' if the internal resistance 'r' is very small in comparison with 'R'.

§ Ideal constant current source:

The current source whose output current remains constant whatever the change in the load resistor is called ideal constant current source. Such current source possesses infinite or very internal resistance in comparison with the load resistance. In practice, none such ideal constant current source can be constructed.

Fig. 1a shows a resistor 'R' is connected across a current source. The internal resistance of a current source is always in parallel with the source. The equivalent circuit is shown in fig. 1b in which r indicates the internal resistance of the source. Therefore, the current through the resistor R is $\frac{rI}{r+R}$ and can be close to 'I' if the internal resistance 'r' is very high in comparison with 'R'.

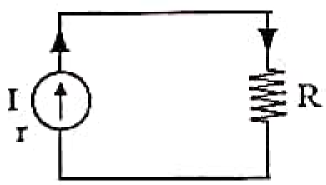

Fig. 1a.

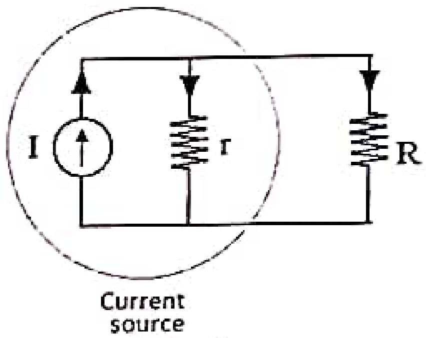

Fig. 1b.

§ Superposition theorem:

It states that in any linear circuit containing multiple independent energy sources, the current which flows at any point is the algebric sum of all the currents which would flow at that point if each source is considered separately and all the other sources are replaced by their internal resistances.

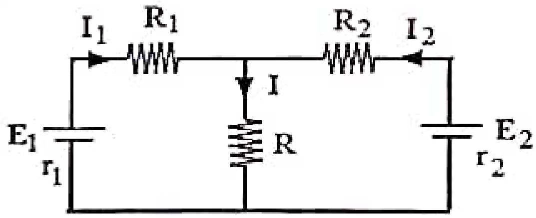

Fig. 1.

Consider the linear circuit as shown in fig. 1 in which I_1, I_2, I_3 represent the currents due to simultaneous action of the two voltage sources. To obtain current at any point in the circuit due to Source E_1 alone, we replace the source E_2 by its internal resistance r_2 and redraw the circuit as shown in fig. 2.

$$I_1' = \frac{E}{r_1 + R_1 + \left(\frac{R(R_2 + r_2)}{R + (R_2 + r_2)}\right)} \qquad (1)$$

$$I_2' = \frac{I_1'R}{R + (R_2 + r_2)} \qquad (2)$$

$$I' = \frac{I_1'(R_2 + r_2)}{R + (R_2 + r_2)} \qquad (3)$$

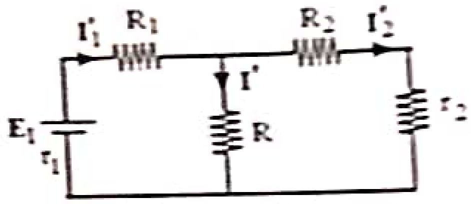

Fig. 2.

Similarly to obtain current at any point in the circuit due to Source E_2 alone, we replace the source E_1 by its internal resistance r_1 and redraw the circuit as shown in fig. 3.

$$I_2'' = \frac{E_2}{r_2 + R_2 + \left(\frac{R(R_1 + r_1)}{R + (R_1 + r_1)}\right)} \qquad (4)$$

$$I_1'' = \frac{I_2'R}{R + (R_1 + r_1)} \qquad (5)$$

$$I'' = \frac{I_2''(R_1 + r_1)}{R + (R_1 + r_1)} \qquad (6)$$

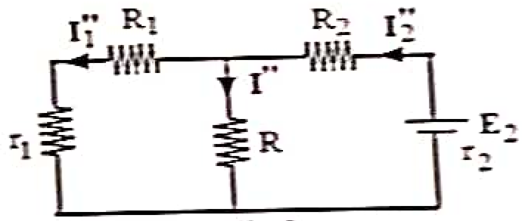

Fig. 3.

By combining the current values of fig. 2 and fig. 3, the actual current values in fig. 1 due to simultaneous action of both sources are

$$I_1 = I_1' - I_1'' \qquad (7)$$

$$I_2 = I_2'' - I_2' \qquad (8)$$

$$I = I' + I'' \qquad (9)$$

§ Thevenin's Theorem:

Any linear electric network with current and voltage sources can be replaced by an equivalent circuit containing a single independent voltage source V_{TH} and a series resistance R_{TH}.

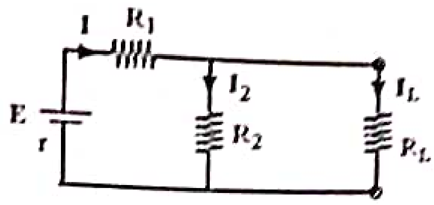

Fig. 1.

Consider the circuit as shown in fig. 1. Following are the simple steps to analyze electric circuit through Thevenin's Theorem.

1. Open the load resistor (fig. 2).
2. Calculate the open circuit voltage. This is the Thevenin Voltage (V_{TH}).

$$V_{TH} = \frac{ER_2}{r + R_1 + R_2} \qquad (1)$$

3. Replace energy sources by their internal resistances (fig. 3).
4. Calculate the open circuit resistance. This is the Thevenin Resistance (R_{TH}).

$$R_{TH} = \frac{(r + R_1)R_2}{(r + R_1) + R_2} \qquad (2)$$

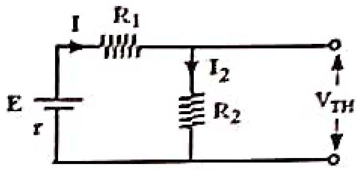

Fig. 2.

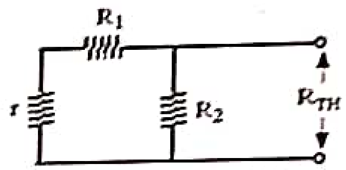

Fig. 3.

5. Now, redraw the circuit with measured open circuit voltage (V_{TH}) in Step (2) as voltage source and measured open circuit resistance (R_{TH}) in step (4) as a series resistance and connect the load resistor which we had removed in Step (1). This is the equivalent Thevenin Circuit (fig. 4) of the given Linear Electric Network.

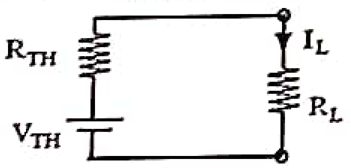

Fig. 4.

6. Now find the load current flowing through Load resistor by using the Ohm's Law

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} \qquad (3)$$

§ Maximum Power transfer Theorem:

For any power source, the power transferred from the power source to the load is maximum when the resistance of the load R_L is equal to the internal resistance R_{in} of the source. The process used to make $R_L = R_{in}$ is called impedance matching.

Proof: Consider the circuit as shown in fig. 1.

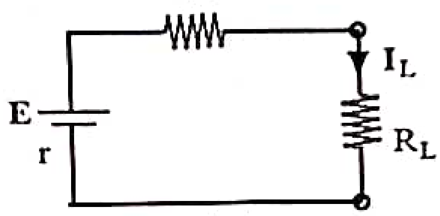

Fig. 1.

The internal resistance of the network is

$$R_{in} = r + R \tag{1}$$

Therefore current through the load resistance is

$$I_L = \frac{E}{R_{in} + R_L} \tag{2}$$

The voltage drop across load is

$$V_L = R_L I_L = \frac{R_L E}{R_{in} + R_L} \tag{3}$$

Therefore, the power supplied to the load by the source is

$$P_L = V_L I_L = \frac{R_L E}{R_{in} + R_L} \frac{E}{R_{in} + R_L} = \frac{R_L E^2}{(R_{in} + R_L)^2} \tag{4}$$

If this power is to be maximum with respect to R_L then we must have

$$\frac{dP_L}{dR_L} = 0 \tag{5}$$

Thus differentiating equation (4) with respect to R_L we get

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L} \frac{R_L E^2}{(R_{in} + R_L)^2} = \frac{E^2}{(R_{in} + R_L)^2} + R_L E^2 \frac{d}{dR_L}(R_{in} + R_L)^{-2}$$

or $$\frac{dP_L}{dR_L} = \frac{E^2}{(R_{in} + R_L)^2} + \frac{R_L E^2 (-2)}{(R_{in} + R_L)^3} \frac{d}{dR_L}(R_{in} + R_L)$$

or $$\frac{dP_L}{dR_L} = \frac{E^2}{(R_{in} + R_L)^2} + \frac{R_L E^2 (-2)}{(R_{in} + R_L)^3} = \frac{E^2 (R_{in} + R_L - 2R_L)}{(R_{in} + R_L)^3}$$

or $$\frac{dP_L}{dR_L} = \frac{E^2 (R_{in} - R_L)}{(R_{in} + R_L)^3} = 0$$

For maximum power we put $\frac{dP_L}{dR_L} = 0$ which gives $R_{in} - R_L = 0$

or $$R_{in} = R_L \tag{6}$$

Hence the proof.